UNIT – IV

8. (a) Find the volume of solid generated by revolving of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(b) Evaluate $\iint e^{(2x+3y)}\,dx\,dy$ over the triangle bounded by $x=0$, $y=0$ and $x+y=1$.

9. (a) Change the order of integration in $I=\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.

(b) Evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$ by changing to polar co-ordinates.

Roll No. ....

3009

B. Tech. 1st Semester (Bio-Tech.) Examination – February, 2022

MATH - I (Series, Matrices and Calculus)

Paper : BSC-MATH-105-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting **one** question from each Unit. Question No. 1 is **compulsory**. All questions carry equal marks.

1. (a) Examine the convergence of the series :

$$1+\frac{1}{4^{2/3}}+\frac{1}{9^{2/3}}+\frac{1}{16^{2/3}}+\ldots\ldots\infty$$

(b) Discuss the convergence of the series :

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{9}+\ldots\ldots$$

(c) Find the rank of the matrix $\begin{bmatrix}1 & 2 & 3\\ 1 & 4 & 2\\ 2 & 6 & 5\end{bmatrix}$

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(d) If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, then evaluate $\frac{d^2y}{dx^2}$.

(e) Explain homogeneous function and state Euler's theorem.

(f) Evaluate $\int_0^{\pi/2} \sin^2 x \cos^{5/2} x\, dx$.

UNIT – I

2. (a) Examine the convergence or divergence of the series : $1+\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\dots\dots\infty,\ x>0$.

(b) Discuss the convergence of the series :

$$1+x+\frac{1}{2}\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+\dots\dots\infty,\ x>0$$

3. (a) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(n+1)^n x^n}{n^{n+1}}$, $x>0$.

(b) Show that the series $\frac{\log 2}{2^2}-\frac{\log 3}{3^2}+\frac{\log 4}{4^2}\dots\dots\infty$ converges.

UNIT – II

4. (a) Reduce the following matrix in normal form and hence find their rank $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.

(b) Are the following vectors linearly dependent ? If so, then find relation between them :

$x_1 = (1, 1, -1, 1)$, $x_2 = (1, -1, 2, -1)$, $x_3 = (2, 1, 0, 1)$.

5. (a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

(b) Use Cayley-Hamilton theorem, find A^{-1}, where

$$A=\begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

UNIT – III

6. (a) If $y = x^2 e^x$, then show that

$$\frac{d^n y}{dx^n}+n(n-2)\frac{dy}{dx}=\frac{1}{2}(n-1)\left\{n\frac{d^2y}{dx^2}+(n-2)y\right\}$$

(b) Calculate the approximate value of $\sqrt{10}$ to four decimal places by Taylor's series.

7. (a) If $u=\log(x^3+y^3+z^3-3xyz)$, then find $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = ?$

(b) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=-\frac{\sin u\cos 2u}{4\cos^3 u}$$